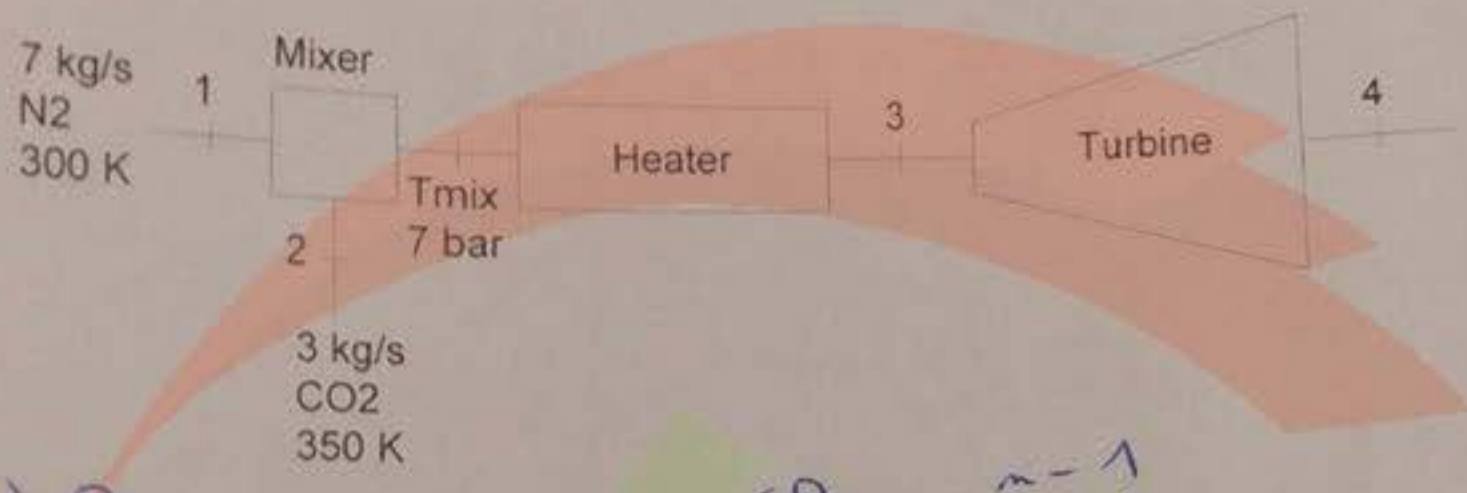
Problem # 1 (50 points):

A mixture of 3 kg/s of carbon dioxide CO_2 at P_2 = 7 bar and 350 K and 7 kg/s of nitrogen N_2 at P_1 = 7 bar and 300 K are mixed and reach T_{mix}. The mixture is heated in a heat exchanger until 500 K at P_3 = 7 bar than expand in a turbine to a pressure P_4 = 1 bar in a polytropic process for which n = 1.271. Determine:

a) The outlet temperature of the turbine T_4 in K.
b) The produced work in kW.
c) The heat transfer of the turbine in kW (No interpolation is required).
d) The heat amount in kW of the heater from T_{mix} up to 500 K. (No interpolation is required).
e) The mixture temperature T_{mix} in K (Iterations are required).



a) $\frac{T_3}{P_3} = \frac{T_4}{P_4}$ $\quad \frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}}$

$\frac{T_4}{500} = \left(\frac{1}{7}\right)^{\frac{n-1}{n}} = \left(\frac{1}{7}\right)^{\frac{1,271-1}{1,271}}$

$T_4 = 500 \times \left(\frac{1}{7}\right)^{\frac{1,271-1}{1,271}} = 330,10\ K$

b) $w = \frac{P_4V_4 - P_3V_3}{1-n} = m\frac{R}{M}\frac{(T_4 - T_3)}{1-n}$

$M = g$

$\frac{\dot{w}}{\dot{m}} = \int_1^2 v\,dP$

$= -\frac{n}{n-1}(P_2v_2 - P_1v_1)$

$\dot{m}_{v_1} = \dot{m}_{a_2}$

$\dot{m}_{v_1} + \dot{m}_w = \dot{m}_{v_2}$

$\dot{m} w_0 = \dot{m}_{v_1} - \dot{m}_{v_2} = \dot{m}_a (w_1 - w_2)$.

$w_1 = 0.622 \left(\frac{0.01258}{1 - 0.01258}\right) = 0.00792$.

$w_2 = 0.622 \left(\frac{0.01072}{1 - 0.01072}\right) = 6.74 \times 10^{-3}$.

$\dot{m}_{a_1} = \frac{V}{v_{a_1}} = \frac{250 \times 28.97 \times 0.9874 \times 10^5}{8314 \times (37+273)} = 277.46 kg/h$

$v_{a_1} = \frac{RT}{M_a P}$

$P_{a_1} = P - P_{v_1} = 1 - 0.01258 = 0.9874$ bar.

$\dot{m}_{v_2} = \dot{m}_a w_2$.

$w_2 = 0.622 \left(\frac{0.02339}{1 - 0.02339}\right)$

$w_2 = 0.015$.

$\dot{m}_{v_2} = \dot{m}_a w_2 = (0.015) 277.46$.

$= 4.16 kg/h \times \frac{1}{3600} \times 10^3$

$\dot{m}_{v_2} = 1.156 g/s$

c) $\dot{m}_{w_5} = \dot{m}_{v_2} - \dot{m}_{v_3}$

$\dot{m}_{a_2} = \dot{m}_{a_3} = \dot{m}_{a_1} = 277.46$ kg/h

$\dot{m}_{w_5} = \dot{m}_a (\omega_2 - \omega_3)$.

$= 277.46 (0.015 - 6.74 \times 10^{-3})$

$= 2.29 \frac{kg}{h} = 0.637$ g/s.

d) $0 = \dot{Q} - \cancel{\dot{W}} + \dot{m}_a [h_{a_2} + \omega_1 h_{v_2}] - \dot{m}_a [h_{a_3} + \omega_3 h_{v_3}]$

$- \dot{m}_{w_5} h_{f(8C)}$.

$\dot{Q} = - \dot{m}_a h_2 + \dot{m}_a . h_3 - \dot{m}_{w_5} h_{f(8C)}$

$h_2 = 1.005 t_2 + \omega_2 (2501.3 + 1.82 t_2)$.

$= 1.005 (20) + 0.015 (2501.3 + 1.82 (20))$.

$= 58.1655$ kJ/kg.

$h_3 = 1.005 (8) + 6.74 \times 10^{-3} (2501.3 + 1.82 (8))$

$= 25$ kJ/kg.

$h_{f(8C)} = 33.6$ kJ/kg.

$\dot{Q} = + 277.46 (25 - 58.1655) - 0.637 (33.6)$

$\dot{Q} = -9223.5$

e)

$$\dot{m}_{a_3} = \dot{m}_{a_4}$$

$$\dot{m}v_3 = \dot{m}v_4 \Rightarrow \omega_3 = \omega_4$$

$$0 = \dot{Q} + \dot{m}_{a_3}(h_{a_3} + \omega_3 h_{v_3}) - \dot{m}_{a_4}(h_{a_4} + \omega h_{v_4})$$

$$\dot{Q} = \dot{m}_a [h_4 - h_3]$$

$$h_4 = 1.005(15) + 6.74\times10^{-3}(2501.3 + 1.82(15))$$

$$= 32.12 \text{ kJ/kg}$$

$$h_3 = 25 \text{ kJ/kg}$$

$$\dot{Q} = 277.46[32.12 - 25]$$

$$\dot{Q} = 1975.5$$

f) $\phi = \frac{P_{v_4}}{P_{g_4}}$

$$\omega_4 = \omega_3 = 0.622 \frac{P_{v_4}}{1 - P_{v_4}}$$

$$6.74\times10^{-3}(1 - P_{v_4}) = 0.622 P_{v_4}$$

$$P_{v_4} = 4.15\times10^{-3}$$

$$\phi = 0.24$$

$$\phi_4 = 24\%$$

c) 1st law : $0 = \dot{Q} - \dot{W} \rightarrow \dot{m}\left[h_3 + \frac{1}{2}V_3^2 - h_2 - \frac{1}{2}V_4^2\right]$

	y_i	$\bar{h}_i$ (kJ/Kmol) 370 K	$\bar{h}_i$ 500 K
CO_2	0.214	10 570	~~8537~~ 17 678
N_2	0.786	8597	14 581
		$\bar{h}_4 = 9805.22$	$\bar{h}_3 = 14601.76$

$\eta_{mix} = 31.434$

$\dot{Q} = \dot{W} - \dot{m}\left[h_3 - h_4\right]$ / $h_3 = \frac{\bar{h}_3}{\eta_{mix}} = 464.52$ kJ/Kg

$\dot{Q} = 2106.32 - \{10\left[464.52 - 311.93\right]$ $h_4 = \frac{\bar{h}_4}{\eta_{mix}} = 311.93$ kJ/Kmol

$\dot{Q} = 580.42$ kW

d) 1st law : $0 = \dot{Q} - 0 + \dot{m}_{CO_2} h_{CO_2 in} + \dot{m}_{N_2} h_{N_2 in} - \left(\dot{m}_{CO_2} h_{CO_2 out} + \dot{m}_{N_2} h_{N_2 out}\right)$

$\bar{h}_{CO_2 in} = 11\,351$ kJ/Kmol $\Rightarrow h_{CO_2 in} = \frac{\bar{h}}{\eta_{CO_2}} = 258$ kJ/Kmol

$\bar{h}_{N_2 in} = 8723$ kJ/Kmol $\rightarrow h_{N_2 in} = \frac{\bar{h}}{\eta_{N_2}} = 311.53$ kJ/Kmol

at 500 K, $\bar{h}_{CO_2 out} = 17678$ kJ/Kmol

$h_{N_2 out} = 14581$ kJ/Kmol